An asteroid, or comet, viewed from Earth will be either bright or faint depending on many quantifiable factors. Of course the size of the body and its reflectivity make a big difference. So does its distance from the sun and earth at the time you see it. The brightness also depends on whether, from Earth, it is fully-illuminated like the full moon, or only partly-illuminated like the crescent moon.

Astronomers can put all of these variables together into one single equation which works pretty well to predict a body's brightness just about anywhere inside the solar system!

The streak in the photo above is the asteroid 1999AN10 (Courtesy Palomar Digital Sky Survey). Orbit data suggest that on August 7, 2027 it will pass within 37,000 kilometers of Earth. The formula for the brightness of the asteroid is given by:

$$AP\pi R^2 = 4\pi d^2 D^2 10^{\frac{2}{5}(M-m)}$$

where: A is the Albedo of the asteroid (A=1 means it's a mirror; A=0 means it is totally non-reflective), P is the phase of the asteroid viewed from Earth (1.0 = fully-illuminated, 0.5= half-illuminated); R is the asteroid radius in kilometers; d is the distance to Earth in kilometers; D is the distance of the asteroid from the sun in kilometers; M is the apparent brightness of the sun at Earth's orbit (M= -26); and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The photograph above shows stars as faint as m = +20.

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 384,000 kilometers, and D = 150 million kilometers, what is the formula R(m) for the asteroid if its albedo is similar that of lunar rock (A = 0.1) and it is observed in nearly full-phase (P=0.9)?

Problem 2 - If the radius of the asteroid is in the domain between 1 kilometer and 100 kilometers, what is the range of apparent brightnesses?

Answer Key

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 384,000 kilometers, and D = 150 million kilometers, what is the formula R(m) for the asteroid if its albedo is similar that of lunar rock (A = 0.1) and it is observed in nearly full-phase (P=0.9)? Answer: Substitute the given values into the equation and simplify. The formula will give the radius of the asteroid in kilometers as a function of its apparent brightness (called apparent magnitude by astronomers).

$(0.1)(0.9)(3.141)\ R(m)^2 = 4\ (3.141)\ (384000)^2\ (150{,}000{,}000)^2\ 10^{0.4(-26\ -\ m)}$

$R(m)^2 = 1.5 \times 10^{29}\ 10^{-10.4}\ 10^{-0.4m}$

$R(m)^2 = 1.5 \times 10^{29}\ 10^{-10.4}\ 10^{-0.4m}$

$R(m)^2 = 6.0 \times 10^{18}\ 10^{-0.4m}$

$R(m) = 2.4 \times 10^{9}\ 10^{-0.4m}$

Problem 2 - If the radius of the asteroid is in the domain between 1 kilometer and 100 kilometers, what is the range of apparent brightnesses? Answer: Solve the formula for R(m) for m(R) and evaluate for R = 1 and R = 100 to obtain the range of the function.

$m(R) = -\ 5/2\ \log_{10}(R/2.4 \times 10^{9})$

so m(R) for R = 1 yields $m(1) = -5/2\ (-9.4)$ so $m(1) = +23$
and $m(100) = -5/2\ (-7.4)$ so $m(100) = +18$

so Domain R: [1,100]
and Range m: [+23, +18]